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DIFFERENTIAL MOTION ANALYSIS OF LAB-VOLT R5150 ROBOT SYSTEM

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ABSTRACT

Differential motion is a way to track and explain motion for different points of the robot. It can be used to study movement of robot mechanisms through a Small period of time. In this paper, forward kinematics modeling and differential motion analysis of (5 DOF) Lab-Volt R5150 robotic manipulator are presented. The standard Denavit-Hartenberg (DH) model is applied to build the mathematical modeling to determine and simulate the position and orientation of the end effector for the 5DOF Lab-Volt R5150 robot manipulator. This position will be used to calculate the required joint variable. Differential motion uses Jacobian method to calculate and analyze the end-effector velocity and its relation to the joint variables velocities. A diagram between (velocities - time) has been presented and drawn for several cases. Important conclusions are reported from the values obtained.

INTRODUCTION

Robot arm kinematics deals with the analytic study of the geometry of motion of a robot arm with respect to a fixed reference co-ordinate system as a function of time without considering the forces and/or moments that cause the motion [1].The transformation between the joint space and the Cartesian space of the robot is very important. Robots are operated with their servo motors in the joint space, whereas tasks are defined and objects are manipulated in the Cartesian space. The kinematics solution of any robot manipulator consists of two problems: forward and inverse kinematics. Forward and inverse kinematics is the fundamental problem of the most importance in the robot manipulator’s position control. Forward kinematics will be determined when the joint variable is known whereas inverse kinematics will be calculated when the position and orientation of the end effector are known [2,3].

Many robotic tasks require that the end-effector location and velocity be controlled at all point along a trajectory. For this manipulator, not only the final location of the end effector is important, but also the velocity with which the end effector would move to reach the final location is also an equally important concern. The conversion from joint velocities to the end effector velocity is described by a matrix called Jacobian matrix, an essential mathematical tool for the analysis and control of a robot's function and operation, which represents the differential relationship between the joint displacements and the resulting end-effecter motion. The Jacobian matrix which is on manipulator configuration is a linear mapping from velocities in joint space to velocities in Cartesian space [4,5]. Jacobian is encountered in many aspects of robotic manipulation: in the planning and execution of smooth trajectories, in the determination of singular configurations, in the execution of coordinated anthropomorphic motion, in the derivation of the dynamic equations of motion, and in the transformation of forces and torques from the end-effector to the manipulator joints [6].

DESCRIPTION OF THE LAB-VOLT R5150 ROBOT MANIPULATOR

Lab-Volt R5150 Robot is a small table top robotic arm with a five articulated coordinate robotic manipulator that uses stepper motors for joint actuators, and its motion are controlled by RoboCIM R5150 software. Lab-Volt R5150 has five rotary joints (base, shoulder, elbow, tool pitch and tool roll) as in figure (1). It is providing five directions of motion (DOF) plus a grip movement [7].The coordinate frame assignment is shown in Figure (2).

Figure 1: Lab-Volt R5150 manipulator

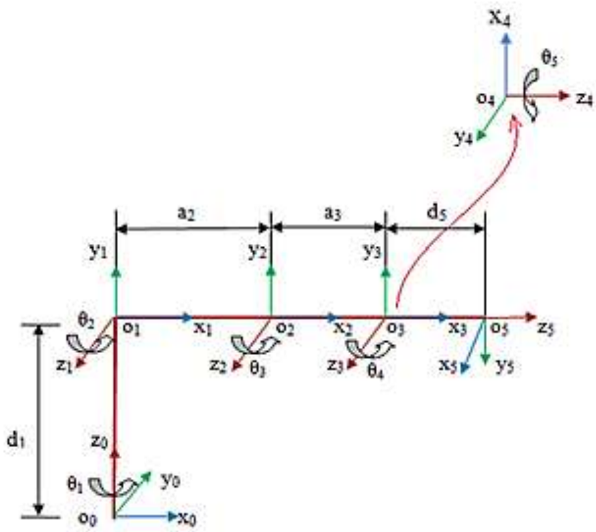

Figure 2: Coordinate frames for Lab-Volt 5150 manipulator

KINEMATIC ANALYSIS OF LAB-VOLT R5150 ROBOTIC ARM

Forward Kinematics

The forward kinematics shows the transformation from one frame into another one, starting at the base and ending at the end-effector. For making this model a convention needs to be chosen for selecting frames of reference for the different links of the robot arm. A commonly used convention for selecting frames of reference in robotic applications is the Denavit-Hartenberg or DH convention [8]. This method was produced by Jaques Denavit and Richard Hartenberg, which is considered as the best method to calculate the forward kinematic as in figure (3). In This method, to set one reference frame relative to another only four parameters are needs instead of six, which are normally required for 3D motion. These parameters are (d_i, a_i, θ_i, and α_i) which tell the location of a link-frame of the robot from a previous link-frame. The transformation matrix between two neighboring frames is expressed as shown in Equation (1).

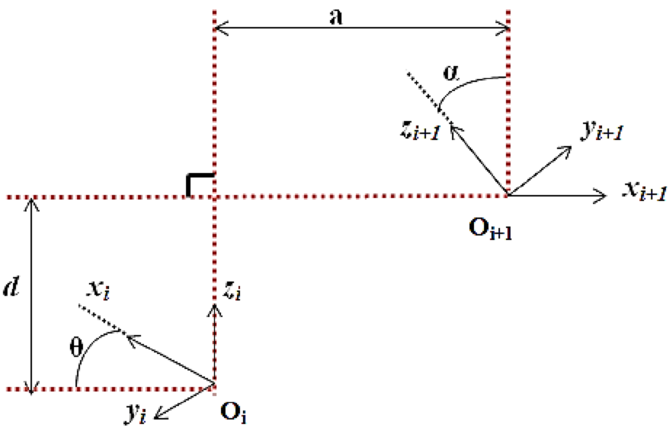

Figure 3: D-H frame assignment

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$A_i = Rot_{z,\theta i}\ Trans_{z,di}\ Trans_{x,ai}\ Rot_{x,\alpha i}$

$$= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i c\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i c\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The parameters for the Lab-Volt 5150 robot arm are listed in the table (1), where shows rotation about the z-axis, rotation about the x- axis, transition along the z-axis, and transition along the x-axis. By substituting the D-H parameters in Table (1) into Eq. (1), we can obtain the individual transformation matrices A_1^0 to A_5^4, and a global matrix of transformation A_5^0, as illustrated in figure (2).

Table 1. D-H parameters for Lab-Volt 5150 arm

Frame	θ_i	d_i (m)	a_i (m)	α_i (degree)
1	θ_1	255.5	0	90
2	θ_2	0	190	0
3	θ_3	0	190	0
4	θ_4	0	0	90
5	θ_5	115	0	0

$$A_1^0 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_2^1 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_3^2 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_4^3 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^4 = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^0 = \begin{bmatrix} c_{12345} + s_{15} & -s_5c_{1234} + s_1c_5 & c_1s_{234} & c_1(d_5s_{234} + a_3c_{23} + a_2c_2) \\ s_1c_{2345} - c_1s_5 & -s_{15}c_{234} - c_{15} & s_1s_{234} & s_1(d_5s_{234} + a_3c_{23} + a_2c_2) \\ c_5s_{234} & -s_5s_{234} & -c_{234} & -d_5c_{234} + a_3s_{23} + a_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Velocity Kinematics

The forward kinematic equation is defined as a function between the positions and orientations of Cartesian space and the positions of joint space. Velocity relationships are then determined by a matrix which relates changes in joint parameter velocities to Cartesian velocities which is called the Jacobian Matrix. The Jacobian is one of the most important quantities in the analysis and control of robot motion. This is a time-varying, position dependent linear transform. It is used for a smooth trajectory planning and execution in the derivation of the dynamic equation. The relationships between joint velocities $\dot{\theta}$ and the linear velocity $\dot{p}$ and angular velocity ω of the end effector:

$$\dot{p} = J_p(\theta)\dot{\theta} \quad (3)$$
$$\omega = J_\omega(\theta)\dot{\theta} \quad (4)$$

Where $\theta = \begin{bmatrix} \theta_1 \\ \cdot \\ \cdot \\ \theta_n \end{bmatrix}$

The above equations can be combined to form J, which include both linear and angular velocity:

$$v = J(\theta)\dot{\theta} \quad (5)$$

Where $J(\theta)$is in the form:

$$J(\theta) = \begin{bmatrix} J_{p1} & & J_{pn} \\ & 6*n & \\ J_{\omega 1} & & J_{\omega n} \end{bmatrix}$$

It has a number of columns equal to the number of degrees of freedom in joint space, and a number of rows equal to the number of degrees of freedom in Cartesian space, three rows for linear velocity in the x, y and z directions, and three for angular velocity.

DETERMINATION OF JACOBIAN MATRIX FOR LAB-VOLT R5150 ROBOT ARM

Jacobian matrix for Lab-Volt R5150 Robot Arm can be determined from the following equations:

$$J_{pi} = \begin{cases} z_{i-1} \times (o_n - o_{i-1}) & \text{for revolute joint i} \\ z_{i-1} & \text{for prismatic joint i} \end{cases} \quad (6)$$

$$J_{\omega i} = \begin{cases} z_{i-1} & \text{for revolute joint i} \\ 0 & \text{for prismatic joint i} \end{cases} \quad (7)$$

Where:

J_{pi} = linear jacobian matrix, $J_{\omega i}$ = angular jacobian matrix.

Then, Jacobian matrix is:

$$J(\theta) = \begin{bmatrix} z_0 \times (o_5 - o_0) & z_1 \times (o_5 - o_1) & z_2 \times (o_5 - o_2) & z_3 \times (o_5 - o_3) & z_4 \times (o_5 - o_4) \\ z_0 & z_1 & z_2 & z_3 & z_4 \end{bmatrix} \quad (8)$$

From forward kinematic, we can find the value of (on - oi-1)

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad o_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \quad o_2 = \begin{bmatrix} a_2c_1c_2 \\ a_2s_1c_2 \\ a_2s_2 + d_1 \end{bmatrix}, \quad o_3 = \begin{bmatrix} a_3c_1c_{23} + a_2c_1c_2 \\ a_3s_1c_{23} + a_2s_1c_2 \\ a_3s_{23} + a_2s_2 + d_1 \end{bmatrix}, \quad o_4 = \begin{bmatrix} 0 \\ 0 \\ d_5 \end{bmatrix},$$

$$o_5 = \begin{bmatrix} d_5c_1s_{234} + a_3c_1c_{23} + a_2c_1c_2 \\ d_5s_1s_{234} + a_3s_1c_{23} + a_2s_1c_2 \\ -d_5c_{234} + a_3s_{23} + a_2s_2 + d_1 \end{bmatrix}$$

Furthermore, the direction of the joint axes ($z_0 \ldots z_4$) can be found from the forward kinematic analysis. From matrix A_1^0 the direction of joint axis z_0 is:

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

From Matrix A_1^0, the value of vector z_1 is:

$$z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

The vector (z_2) is the calculated from multiplying Matrix $A_1^0$with Matrix A_2^1 to get A_2^0:

$$A_2^0 = A_1^0 \,.\, A_2^1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & a_2c_1c_2 \\ s_1c_2 & -s_1s_2 & -c_1 & a_2s_2s_1 \\ s_2 & c_2 & 0 & a_2s_1 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, $$z_2 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

The vector (z_3) is the calculated from multiplying Matrices A_1^0, A_2^1 and A_3^2:

$$A_3^0 = A_1^0 .\ A_2^1 .\ A_3^2 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} c_{123} & -c_1s_{23} & s_1 & c_1(a_3c_{23} + a_2c_2) \\ s_1c_{23} & -s_{123} & -c_1 & s_1(a_3c_{23} + a_2c_2) \\ s_{23} & c_{23} & 0 & a_3s_{23} + a_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, $$z_3 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

Matrix A_3^0 is then multiply by matrix A_4^3 to obtain:

$A_4^0 = A_3^0\ A_4^3$

$$= \begin{bmatrix} c_{123} & -c_1s_{23} & s_1 & c_1(a_3c_{23} + a_2c_2) \\ s_1c_{23} & -s_{123} & -c_1 & s_1(a_3c_{23} + a_2c_2) \\ s_{23} & c_{23} & 0 & a_3s_{23} + a_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1c_{234} & s_1 & c_1s_{234} & c_1(a_3c_{23} + a_2c_2) \\ s_1c_{234} & -c_1 & s_1s_{234} & s_1(a_3c_{23} + a_2c_2) \\ s_{234} & 0 & -c_{234} & a_3s_{23} + a_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, $$z_4 = \begin{bmatrix} c_1s_{234} \\ s_1s_{234} \\ -c_{234} \end{bmatrix}$$

Then the elements of the Jacobian matrix of the five axis articulated robot will become:

$$z_0 \times (o_5 - o_0) = \begin{bmatrix} -d_5s_1s_{234} - a_3s_1c_{23} - a_2s_1c_2 \\ d_5c_1s_{234} + a_3c_1c_{23} + a_2c_1c_2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1 \times (o_5 - o_1) = \begin{bmatrix} d_5c_1c_{234} - a_3c_1s_{23} - a_2s_2c_1 \\ d_5s_1c_{234} - a_3s_1s_{23} - a_2s_2s_1 \\ d_5s_{234} + a_3c_{23} + a_2c_2 \\ s_1 \\ -c_1 \\ 0 \end{bmatrix},$$

$$z_2 \times (o_5 - o_2) = \begin{bmatrix} d_5c_1c_{234} - a_3c_1s_{23} \\ d_5s_1c_{234} - a_3s_1s_{23} \\ d_5s_{234} + a_3c_{23} \\ s_1 \\ -c_1 \\ 0 \end{bmatrix}, \quad z_3 \times (o_5 - o_3) = \begin{bmatrix} d_5c_1c_{234} \\ d_5s_1c_{234} \\ d_5s_{234} \\ s_1 \\ -c_1 \\ 0 \end{bmatrix}, \quad z_4 \times (o_5 - o_4) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ c_1s_{234} \\ s_1s_{234} \\ -c_{234} \end{bmatrix}$$

Now the final form of the Jacobian matrix will become as shown below:

$$J = \begin{bmatrix} -d_5s_1s_{234} - a_3s_1c_{23} - a_2s_1c_2 & d_5c_1c_{234} - a_3c_1s_{23} - a_2s_2c_1 & d_5c_1c_{234} - a_3c_1s_{23} & d_5c_1c_{234} & 0 \\ d_5c_1s_{234} + a_3c_1c_{23} + a_2c_1c_2 & d_5s_1c_{234} - a_3s_1s_{23} - a_2s_2s_1 & d_5s_1c_{234} - a_3s_1s_{23} & d_5s_1c_{234} & 0 \\ 0 & d_5s_{234} + a_3c_{23} + a_2c_2 & d_5s_{234} + a_3c_{23} & d_5s_{234} & 0 \\ 0 & s_1 & s_1 & s_1 & c_1s_{234} \\ 0 & -c_1 & -c_1 & -c_1 & s_1s_{234} \\ 1 & 0 & 0 & 0 & -c_{234} \end{bmatrix} \quad (9)$$

RESULTS AND DISCUSSION

Kinematic analysis of Lab-Volt R5150 robot have been applied for several cases to calculate the final position and orientation theoretically and compared with RoboCIM software results, as illustrated in table (2). It was noticed that the results of the experimental values and simulation values in table (2) are very close to each other and the percentage error between them ranging for X-axis from (0.047 – 0.115) %, Y-axis from (0.031 – 0.117) % and Z-axis from (0.025 – 0.6)%. After that the velocities these cases are plotted with time as shown in figure (4).

Table 2. The output data generated manually and virtually

Cases	Joint Angles	End-effector position (True)	End-effector position (Measured)	Error%= $\frac{\text{True}-\text{Measured}}{\text{True}}*\%$
1	θ1 = 10 θ2 = 37 θ3 = -76.56 θ4 = 13.01 θ5 =3	Px = 243.070 Py = 42.860 Pz = 145.964	Px = 243.35 Py = 42.91 Pz = 146	0.115 % 0.117% 0.025 %
2	θ1 = 45 θ2 = 14.78 θ3 = 20.48 θ4 = 54.73 θ5 =45.02	Px = 320.925 Py = 320.925 Pz = 413.635	Px = 321.20 Py = 321.20 Pz = 414.10	0.085 % 0.085% 0.112 %
3	θ1= -32.98 θ2 = 20.64 θ3 = 45.79 θ4 = 30.9 θ5 = -280.73	Px = 308.566 Py = -200.232 Pz = 511.295	Px = 308.71 Py = -200.32 Pz = 511.98	0.047 % 0.044% 0.134 %

4	θ1= 95.04 θ2 = -15.16 θ3 = -65.98 θ4 = 75.91 θ5 = 180.17	Px = -17.761 Py = 201.387 Pz = -96.442	Px = -17.77 Py = 201.45 Pz = -97.02	0.051 % 0.031% 0.6 %

Jacobian method will be applied and estimated according to equation (9) for the taken cases to obtain the value of the velocity for each joint.

Case (1): (θ_1 = 10, θ_2 = 37, θ_3 = -76.56, θ_4 = 13.01 and θ_5 =3).

$$J = \begin{bmatrix} -42.86 & 107.872 & 220.48 & 101.31 & 0 \\ 243.07 & 19.021 & 38.876 & 17.864 & 0 \\ 0 & 246.82 & 95.08 & -51.402 & 0 \\ 0 & 0.174 & 0.174 & 0.174 & -0.44 \\ 0 & -0.985 & -0.985 & -0.985 & -0.078 \\ 1 & 0 & 0 & 0 & -0.894 \end{bmatrix}$$

Case (2): (θ_1 = 45, θ_2 = 14.78, θ_3 = 20.48, θ_4 = 54.73 and θ_5 =45.02)

$$J = \begin{bmatrix} -320.925 & -111.818 & -77.544 & 0.014 & 0 \\ 320.925 & -111.818 & -77.544 & 0.014 & 0 \\ 0 & 453.856 & 270.143 & 115 & 0 \\ 0 & 0.707 & 0.707 & 0.707 & 0.707 \\ 0 & -0.707 & -0.707 & -0.707 & 0.707 \\ 1 & 0 & 0 & 0 & -1.7 \times 10^{-4} \end{bmatrix}$$

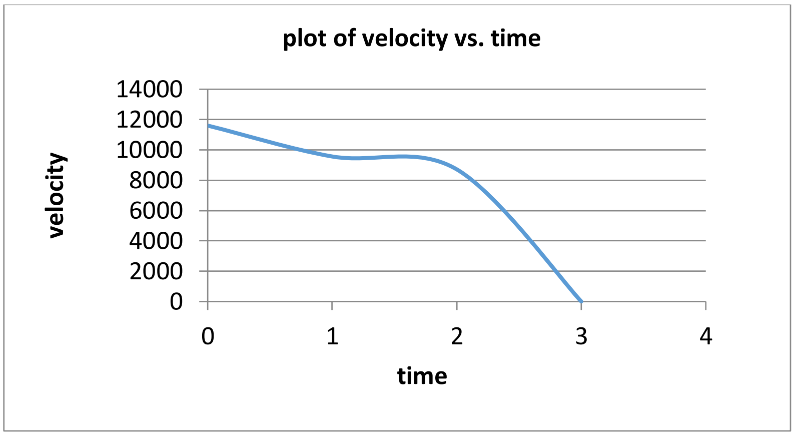
Case (3): (θ_1= -32.98, θ_2 = 20.64, θ_3 = 45.79, θ_4 = 30.9 and θ_5 = -280.73)

$$J = \begin{bmatrix} 103.445 & -214.576 & -158.394 & -12.308 & 0 \\ 308.566 & 139.241 & 102.784 & 7.987 & 0 \\ 0 & 367.84 & 190.035 & 114.06 & 0 \\ 0 & -0.544 & -0.544 & -0.544 & 0.832 \\ 0 & -0.839 & -0.839 & -0.839 & -0.54 \\ 1 & 0 & 0 & 0 & 0.127 \end{bmatrix}$$

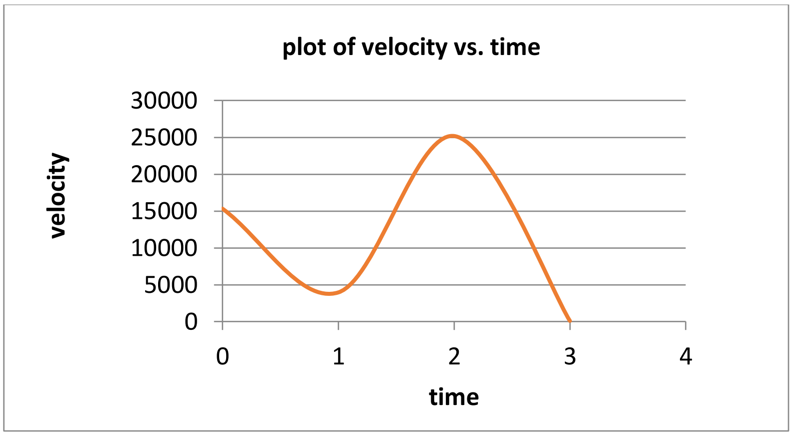
Case (4): (θ_1= 95.04, θ_2 = -15.16, θ_3 = -65.98, θ_4 = 75.91 and θ_5 = 180.17)

$$J = \begin{bmatrix} -201.387 & -30.918 & -26.553 & -10.061 & 0 \\ -17.761 & 350.581 & 301.085 & 114.078 & 0 \\ 0 & 202.169 & 18.781 & -113.628 & 0 \\ 0 & 0.996 & 0.996 & 0.996 & 8.01 \times 10^{-3} \\ 0 & 0.088 & 0.088 & 0.088 & -0.091 \\ 1 & 0 & 0 & 0 & -0.996 \end{bmatrix}$$

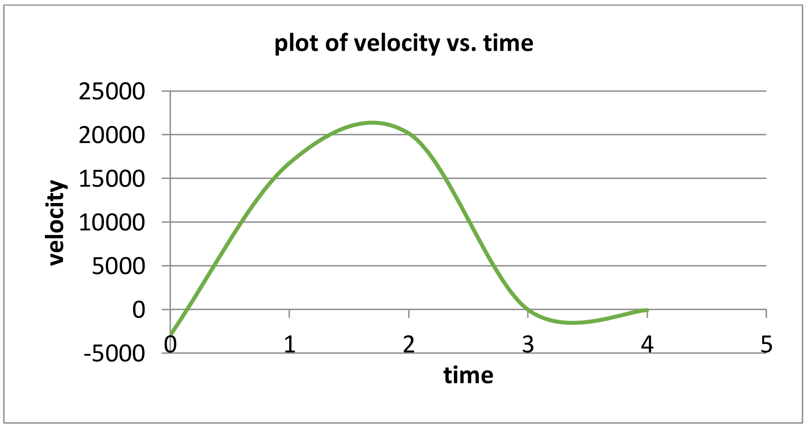
The resulting Jacobin matrix for each case has been product with the joint velocity according to equation (5) to produce the required velocity of the end-effector as bellow:



Case (1)



Case (2)



Case (3)

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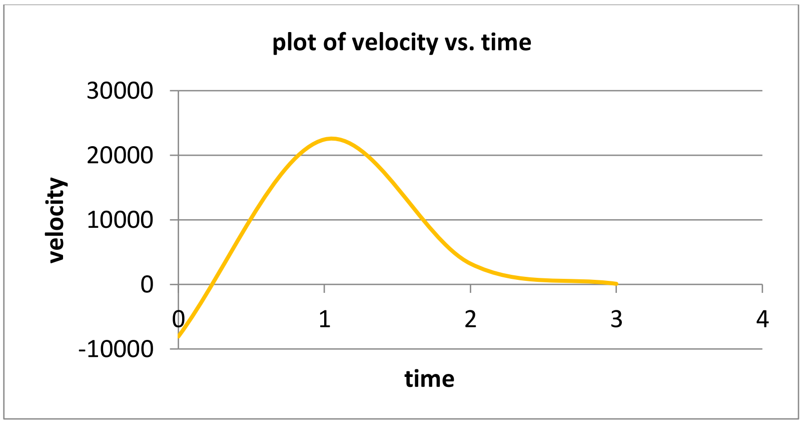

Case (4)

Figure 4: Joint velocities of Lab-Volt R5150 robot arm for case (1, 2, 3 and 4) when $\dot{\theta}$ = 30° /s.

CONCLUSION

From table (2), the value of the errors in z-axis is greater than other axis resulting from the contents of the servo motor and the weight of the arm. To reduce or eliminate this error, dynamic modeling should be followed.
Jacobian matrix was calculated for the three cases and used to estimate the velocity for the end-effector. The resulting values of the velocity have been invested to estimate the time required to move the robot. It can be observed from the drawn joint velocities in figure (4) that shoulder joint which is joint 2 is reach to the maximum of the curve in case 2, this mean that shoulder joint is very important to control the movement of the robot manipulator.

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